AMRITA VIDYALAYAM

AMRITA PRE BOARD EXAMINATION 2019 - 20

Class : XII **Marks : 80**

Time : 3 hrs

MATHEMATICS

General instructions

1. ***All questions are compulsory.***
2. ***This question paper consists of 36 questions divided into four section A, B, C and D. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.***
3. ***There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.***
4. ***Use of calculator is not permitted.***

SECTION - A

1. The value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is _____.
   a) $\pi/2$ b) $-\pi/2$ c) $\pi/6$ d) $-\pi/6$
2. If $\int_0^1 (3x^2 + 2x + k)\,dx = 0$, then the value of k is _____.
   a) -3 b) -2 c) 1 d) 4
3. Find the value of x if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$
   a) 0 b) 2 c) 1 d) -1
4. For any vector $\vec{a}$, $\vec{a} \times \vec{a}$ is _____.
   a) $\vec{0}$ b) 0 c) $|\vec{a}|\,0$ d None of these
5. The sine of the angle between the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is _____.
   a) $\frac{10}{6\sqrt{5}}$ b) $\frac{4}{5\sqrt{2}}$ c) $\frac{2\sqrt{3}}{5}$ d) $\frac{\sqrt{2}}{10}$
6. If θ is the angle between any 2 vectors a and b then $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to _____.
   a) 0 b) $\pi/4$ c) $\pi/2$ d) π
7. The determinant of $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$ is _____.
   a) 1 b) 0 c) 1/2 d) $\sqrt{3}/2$
8. If a line makes angles 90^0 and 60^0 respectively with the positive directions of x and y axes, then the angle which it makes with the positive direction of z axis is _____.
   a) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ b) $\frac{\pi}{4}$ or $\frac{5\pi}{6}$ c) $\frac{-\pi}{6}$ or $\frac{4\pi}{6}$ d) $\frac{\pi}{6}$ or $\frac{-5\pi}{6}$
9. If $P(A \cap B) = 0.15$, $P(\overline{B}) = 0.10$ then P(A/B) is equal to _____.
   a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{6}$ d) $\frac{1}{5}$
10. If $2P(A) = P(B) = 5/13$ and $P(A/B) = 2/5$, then $P(A \cup B)$ is _____.
    a) $\frac{11}{26}$ b) $\frac{26}{11}$ c) $\frac{3}{5}$ d) $\frac{4}{7}$

11. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$ then $(fog)(7) =$ _____.
12. If $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$, then $(A + A') =$ _____.
13. The area enclosed between the graph $y = x^3$ and the line $x = 0$, $y = 1$, $y = 8$ is _____.

OR

The area enclosed between the graph of $y = 2x - x^2$ and x axis is _____.

14. The value of k for which the following function $f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$ is _____.
15. The area of the parallelogram whose adjacent sides are $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is _____.

OR

If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$ and $|\vec{a}| = 5$, then the value of $|\vec{b}| =$ _____.

16. If $A = \begin{vmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$, then what is the co factor of the element a_{21}?
17. Find $\int \frac{dx}{x^2 + 16}$.
18. Find $\int (1 - x)\sqrt{x}\, dx$.
19. Write the integrating factor of the following differential equation.
    $(1 + y^2)dx - (\tan^{-1} y - x)\, dy = 0$
20. Find $\int \frac{3x}{3x - 1}\, dx$.

OR

Evaluate $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1 - x^2}}\, dx$.

SECTION - B

21. Show that the relation R defined by $(a, b)\ R\ (c, d) \Rightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

OR

Prove that $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3)$, where $x \in [-1/2, 1/2]$.

22. Differentiate $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$ with respect to x.
23. Find the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where a and b are arbitrary constants.
24. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

OR

Find the direction cosines of the vector joining the points A(1, 2, -3) and B(-1, -2, 1) directed from A to B.

25. Find the co-ordinates of the foot of the perpendicular drawn from the origin on the line.
    $\frac{x - 4}{-1} = \frac{y - 1}{3} = \frac{z - 3}{-2}$
26. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

SECTION - C

27. If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find dy/dx when $\theta = \pi/3$.

OR

If $y = a(\sin t - t \cos t)$ and $x = a(\cos t + t \sin t)$ find d^2y/dx^2 at $t = \pi/4$.

28. Find $\int \frac{x}{(x^2 + 1)(x - 1)} dx$.

29. Find the particular solution of the differential equation $dy/dx - 3y \cot x = \sin 2x$, given that $y = 2$, when $x = \pi/2$.

30. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 17.50 per package on nuts and ₹ 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit if he operates his machines for at the most 12 hours a day? From the above as a linear programming problem and solve it graphically.

31. Of the students in a college, it is known that 70% reside in hostel and 30% reside outside hostel. Previous year results report that 40% of the hostelers attain A grade and 20% of those who reside outside attain A grade in the annual examination. At the end of the year a student is chosen at random from the college and he has an A grade. What is the probability that the student is a hosteler?

OR

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

32. Let $A = R - \{3\}$, $B = R - \{1\}$, let $f : A \to B$ be defined by $f(x) = \frac{x - 2}{x - 3}$, for all $x \in A$, show that f is bijective, also find

a) x if $f^{-1}(x) = 4$ b) $f^{-1}(7)$

SECTION - D

33. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A-1 and then solve the system of equations.

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

OR

Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

34. Using the method of integration, find the area of the region bounded by the triangle ABC whose vertices are A(1, 2), B(2, 0) and C(4, 3).

35. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

OR

Show that the semi vertical angle of a cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

36. Find the distance between the point (7, 2, 4) and the plane determined by the points A(2, 5, -3), B(-2, -3, 5) and C(5, 3, -3).